Math 350 Final Exam Topics

Here is the broad list of topics which will be on the final exam. The exam will be approximately 25% differentiation, 25% integration and 50% vector calculus theorems. Below each topic is a list of suggested practice problems from the book, in the form page(problem, problem, ...). All problems are odd numbered, and answers are in the back of the book. You need not do all of these, but you should make sure that you are confident that you're *able* do to all of these. The last page contains the list of formulas that will be available to you on the exam. Any other formulas you think you will need should be memorized.

I. Differentiation

- Definition of the total derivative, special cases for curves (velocity) and scalar functions (gradient). (There will be no questions about whether derivatives exist, are continuous, etc. However you should understand the definition).

$$140(7,13) \quad 149(5,7,9,11) \quad 174(3,21,39)$$

- The Chain Rule

$$159(3,5,7(\text{use the chain rule to prove the product rule})) \quad 174(39,45)$$

- Taylor Expansions up to 2nd order of scalar functions (explicit form of the remainder term is not required.)

$$202(1,3,5)$$

- Gradient test and second derivative test for local extrema; Global extrema & constrained extrema using Lagrange multipliers. (Only the second derivative test for unconstrained extrema is required.)

$$222(1,3,5,23,25) \quad 243(1,3,5,13,15,29)$$

II. Integration

- Definition of 2D/3D integrals using Riemann Sums, and how you compute them in practice (iterated integrals) using Fubini's Theorem. (No fiddly questions about integrability, but I would like you to understand the conditions that allows you to use Fubini's Theorem). Integrating over regions bounded by graphs (simple/elementary, etc).

$$347(1,7,11,15) \quad 366(9,21,25,27,29)$$

- Change of variables in 2D/3D

$$391(5,17,19,31) \quad 418(13,21,23)$$

- Line integrals of scalar functions (unoriented) and vector fields (oriented – work, or flux in 2D)

$$427(3,9,11) \quad 447(1,3,15)$$

- Surface integrals of scalar functions (unoriented) and vector fields (oriented – flux in 3D)

$$480(1,3,9) \quad 497(3,5,15)$$

III. Vector Calculus Theorems

- Divergence and Curl

$$314(17,19,21)$$

- Green's Theorem (work form) and Stokes' Theorem (relating *work* around a closed curve to the *curl* over a surface/2D region)

$$528(1, 7, 11) \quad 547(3, 7, 19, 23)$$

- Green's Theorem (flux form) and Divergence/Gauss' Theorem (relating *flux* across a closed curve (in 2D)/surface (in 3D) to the *divergence* over a region in 2D (Green's) or 3D (Div/Gauss'))

$$573(1, 3, 5, 11)$$

- Fundamental Theorem of calculus for Line integrals, and characterization of conservative vector fields on simply connected regions. (The analogous characterization of divergence free vector fields as those which come from curls will not be covered.)

$$558(1, 3, 5, 7, 13, 17) \quad 605(3)$$

Formulas: (Parametrizations assumed to be orientation preserving)

$$
\begin{aligned}
d\mathbf{s} &= \hat{\mathbf{T}}\,ds = \mathbf{c}'(t)\,dt = dx\mathbf{i} + dy\mathbf{j}\,(+dz\mathbf{k}) \\
\hat{\mathbf{n}}\,ds &= dy\mathbf{i} - dx\mathbf{j} \\
d\mathbf{S} &= \hat{\mathbf{n}}\,dS = \mathbf{T}_u \times \mathbf{T}_v\,du\,dv \\
\mathbf{T}_u &= x_u\mathbf{i} + y_u\mathbf{j} + z_u\mathbf{k} \\
\mathbf{T}_v &= x_v\mathbf{i} + y_v\mathbf{j} + z_v\mathbf{k} \\
d\mathbf{S} &= (-g_x\mathbf{i} - g_y\mathbf{j} + \mathbf{k})\,dx\,dy && \text{if } z = g(x,y),\ \hat{\mathbf{n}} \text{ pointing up.} \\
ds &= \|d\mathbf{s}\| = \|\mathbf{c}'(t)\|\ dt \\
dS &= \|d\mathbf{S}\| = \|\mathbf{T}_u \times \mathbf{T}_v\|\ du\,dv \\
dV &= \left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right| du\,dv\,dw = \begin{vmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{vmatrix} du\,dv\,dw \\
dA &= \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} du\,dv \\
(x,y) &= (r\cos\theta, r\sin\theta) && dA = r\,dr\,d\theta \\
(x,y,z) &= (r\cos\theta, r\sin\theta, z) && dV = r\,dr\,d\theta\,dz \\
(x,y,z) &= (\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi) && dV = \rho^2\sin\phi\,d\rho\,d\phi\,d\theta \\
\nabla &= \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k} \\
\mathbf{a} \times \mathbf{b} &= \det\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}
\end{aligned}
$$

- Determinant test for positive definiteness:

  $$\det(A_k) > 0,\ \text{for all } k \iff \mathbf{D}^2 f(\mathbf{x}_0) \text{ positive definite}$$

  where $A_k = k \times k$ submatrix of $\mathbf{D}^2 f(\mathbf{x}_0)$.

- Determinant test for negative definiteness:

  $$\det(A_k) < 0,\ k \text{ odd, and } \det(A_k) > 0,\ k \text{ even} \iff \mathbf{D}^2 f(\mathbf{x}_0) \text{ negative definite}$$

  where $A_k = k \times k$ submatrix of $\mathbf{D}^2 f(\mathbf{x}_0)$.

Orientation conventions

- For a volume V, ∂V has $\hat{\mathbf{n}}$ pointing out of/away from V.
- For a surface $\mathcal{S}$, $\partial\mathcal{S}$ has direction according to right hand rule: RH palm on $\mathcal{S}$, wrist on $\partial\mathcal{S}$, fingers pointing along $\hat{\mathbf{n}} \implies$ thumb points along (in $\hat{\mathbf{T}}$ direction) $\partial\mathcal{S}$.
- For a 2D region R, ∂R oriented such that R is to the left and $\hat{\mathbf{n}}$ is to the right as one travels in the direction $\hat{\mathbf{T}}$.