Math 350 Problem Set 1 (Part II) (due Friday 9/10 by 3pm)

1. Sketch some level curves ($L_c = \{(x,y)|f(x,y) = c\}$ for some choices of c) and the graph of the following functions.

   (a) (5pts) $f(x,y) = (x^2+y^2)^{1/2}$

   (b) (5pts) $f(x,y) = x/y$

2. (5pts) Sketch the level surfaces ($L_c = \{(x,y,z)|f(x,y,z) = c\}$) for $c = 0$ and $c = 1$ of the function

$$f(x,y,z) = xy + z^2$$

3. Evaluate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the following functions. Is the function differentiable?

   (a) (3pts)

$$f(x,y) = \frac{x}{y} + \frac{y}{x}$$

   (b) (3pts)

$$f(x,y,z) = xyz^2$$

4. For each of the following functions $f : \mathbb{R}^n \to \mathbb{R}$ and curves $\mathbf{c} : \mathbb{R} \to \mathbb{R}^n$, compute the gradient of f and the velocity of $\mathbf{c}$. Use this to compute the derivative of $f \circ \mathbf{c} : \mathbb{R} \to \mathbb{R}$ at $t = 0$.

   (a) (5pts) $n = 2$. $f(x,y) = e^{xy}\sin y$, $\mathbf{c}(t) = (2,t)$.

   (b) (5pts) $n = 3$. $f(x,y,z) = x^2+y^2+z^2$, $\mathbf{c}(t) = (t, \cos t, \sin t)$.

5. (5pts) Points in 3 dimensional space $\mathbb{R}^3$ can be described by not only **cartesian coordinates** (x,y,z), but also by **spherical coordinates** (ρ, θ, ϕ) (which we'll cover in much detail later in the term). We can write the former in terms of the latter by

$$\begin{aligned} x &= \rho\cos\theta\sin\phi \\ y &= \rho\sin\theta\sin\phi \\ z &= \rho\cos\phi, \end{aligned}$$

   which can be thought of as a function $[0,\infty) \times [0,2\pi] \times [0,\pi] \subset \mathbb{R}^3 \to \mathbb{R}^3$, $(\rho,\theta,\phi) \mapsto (x,y,z) = (\rho\cos\theta\sin\phi, \rho\sin\theta\sin\phi, \rho\cos\phi)$. Suppose we have a scalar function $f : \mathbb{R}^3 \to \mathbb{R}$, $(x,y,z) \mapsto f(x,y,z)$. Write the expressions for $\frac{\partial f}{\partial \rho}$, $\frac{\partial f}{\partial \theta}$, and $\frac{\partial f}{\partial \phi}$ in terms of $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$ using the chain rule.