Math 350 Problem Set 1 (Part II) (due Friday 9/10 by 3pm)

1. Sketch some level curves \( L_c = \{ (x, y) | f(x, y) = c \} \) for some choices of \( c \) and the graph of the following functions.
   (a) (5pts) \( f(x, y) = (x^2 + y^2)^{1/2} \)
   (b) (5pts) \( f(x, y) = x/y \)

2. (5pts) Sketch the level surfaces \( L_c = \{ (x, y, z) | f(x, y, z) = c \} \) for \( c = 0 \) and \( c = 1 \) of the function \( f(x, y, z) = xy + z^2 \)

3. Evaluate \( \frac{\partial f}{\partial x} \) and \( \frac{\partial f}{\partial y} \) for the following functions. Is the function differentiable?
   (a) (3pts) \( f(x, y) = \frac{x}{y} + \frac{y}{x} \)
   (b) (3pts) \( f(x, y, z) = xyz^2 \)

4. For each of the following functions \( f : \mathbb{R}^n \to \mathbb{R} \) and curves \( \mathbf{c} : \mathbb{R} \to \mathbb{R}^n \), compute the gradient of \( f \) and the velocity of \( \mathbf{c} \). Use this to compute the derivative of \( f \circ \mathbf{c} : \mathbb{R} \to \mathbb{R} \) at \( t = 0 \).
   (a) (5pts) \( n = 2, f(x, y) = e^{xy} \sin y, \mathbf{c}(t) = (2, t) \).
   (b) (5pts) \( n = 3, f(x, y, z) = x^2 + y^2 + z^2, \mathbf{c}(t) = (t, \cos t, \sin t) \).

5. (5pts) Points in 3 dimensional space \( \mathbb{R}^3 \) can be described by not only cartesian coordinates \( (x, y, z) \), but also by spherical coordinates \( (\rho, \theta, \phi) \) (which we’ll cover in much detail later in the term). We can write the former in terms of the latter by:
   \[
   x = \rho \cos \theta \sin \phi \\
   y = \rho \sin \theta \sin \phi \\
   z = \rho \cos \phi,
   \]
   which can be thought of as a function \( [0, \infty) \times [0, 2\pi] \times [0, \pi] \subset \mathbb{R}^3 \to \mathbb{R}^3, (\rho, \theta, \phi) \mapsto (x, y, z) = (\rho \cos \theta \sin \phi, \rho \sin \theta \sin \phi, \rho \cos \phi) \). Suppose we have a scalar function \( f : \mathbb{R}^3 \to \mathbb{R}, (x, y, z) \mapsto f(x, y, z) \). Write the expressions for \( \frac{\partial f}{\partial \rho}, \frac{\partial f}{\partial \theta}, \frac{\partial f}{\partial \phi} \) in terms of \( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \) using the chain rule.