Math 350 Problem Set 2 (Part II) (due Friday 9/17 by 3pm)

1. Compute the total derivative of each function at an arbitrary point (x, y) or (x, y, z).
   (a) (2pts) $f(x, y) = (e^x, \sin xy)$
   (b) (2pts) $f(x, y) = (x + y, x - y, xy)$
   (c) (2pts) $f(x, y, z) = (x + z, y - 5z, x - y)$

2. Find the planes tangent to the following surfaces at the indicated points.
   (a) (3pts) $x^2 + 2y^2 + 3xz = 10$, at $(x, y, z) = (1, 2, 1/3)$
   (b) (3pts) $xyz = 1$, at $(x, y, z) = (1, 1, 1)$

3. Compute the gradient ∇f for the following functions, and find the directional derivative of f in the direction $\mathbf{v}$ at the point p.
   (a) (3pts) $f(x, y, z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$, $\mathbf{v} = \mathbf{i} + \mathbf{k} = (1, 0, 1)$, $p = (1, 1, 1)$.
   (b) (3pts) $f(x, y, z) = xy + yz + xz$, $\mathbf{v} = \mathbf{i} + \mathbf{j} + \mathbf{k} = (1, 1, 1)$, $p = (1, 0, 2)$.

4. (6pts) Compute $g \circ f$, $\mathbf{D}f(x, y)$, $\mathbf{D}g(u, v, w)$ and $\mathbf{D}(g \circ f)(0, 0)$, where
$$f(x, y) = \left(e^x, \cos(y - x), e^{-y}\right), \qquad g(u, v, w) = \left(e^{u-v}, \cos(v + u) + \sin(u + v + w)\right)$$

5. Let $f(x, y) = x^4 y^3 - x^8 + y^4$.
   (a) (3pts) Compute $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y^2}$, and $\frac{\partial^2 f}{\partial y \partial x}$. Verify that $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y}$.
   (b) (3pts) Compute $\frac{\partial^3 f}{\partial x \partial x \partial y}$, $\frac{\partial^3 f}{\partial x \partial y \partial x}$, and $\frac{\partial^3 f}{\partial y \partial x \partial x}$.

6. (6pts) Find the second order Taylor approximation for
$$f(x, y) = e^{-x^2-y^2} \cos(xy)$$
   at $(x_0, y_0) = (0, 0)$.