Math 350 Problem Set 3 (Part II) (due Friday 9/24 by 3pm)

1. Compute the second order Taylor approximation to \( f(x, y) = x^3y + x^2y \) at
   (a) (3pts) \((x_0, y_0) = (0, 0)\). What is \( R_2((0, 0), (x, y)) \)?
   (b) (3pts) \((x_0, y_0) = (1, 1)\).

2. Find the critical points of the following functions and determine whether they are maxima, minima or saddle points.
   (a) (4pts) \( f(x, y) = x^2 + y^2 - xy \)
   (b) (4pts) \( f(x, y) = (x - y)(xy - 1) \).
   (c) (4pts) \( f(x, y, z) = x^2 + y^2 - z^2 \)

3. (6pts) Find all local and global maxima and minima in the region \( x^2 + y^2 \leq 4 \) of the function 
\[
    f(x, y) = (x^2 + y^2)^2 - 2(x^2 + y^2) + 1
\]

4. (6pts) Show that the rectangular parallelepiped (3 dimensional box with 3 pairs of parallel, rectangular sides) with fixed surface area and maximum volume is a cube.