Math 350 Problem Set 4 (due Friday 10/1 by 3pm)

1. (6pts) Find the maximum and minimum of $f(x, y) = xy - y + x - 1$ on the set $x^2 + y^2 \leq 2$.

2. (6pts) Find the point on the intersection of the two planes $a_1 x + a_2 y + a_3 z = 0$ and $b_1 x + b_2 y + b_3 z + b_0 = 0$ that is nearest to the origin $(0, 0, 0)$.

3. (6pts) A firm uses wool and cotton fibers to produce cloth. The amount of cloth produced is given by $Q(x, y) = xy - x - y + 1$, where x is the number of pounds of wool, y the number of pounds of cotton, $x > 1$ and $y > 1$. If wool costs p dollars per pound, and cotton q dollars per pound and the firm can spend B dollars on raw material, what should the ratio of cotton and wool be to produce the most cloth?

4. (6pts) A light ray travels from point A to point B, crossing a boundary between two media (see Figure 3.4.7 on page 245 of the book). In the first medium its speed is v_1 and in the second it is v_2. Show that the trip is made in minimum time when *Snell's law* holds:

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2}$$

(This explains why a pencil or other straight object appears bent when it is partially dipped into water. Also why spear fishing is harder than you might think.)

5. (6pts) Find the points where the equation $f(x, y) = xy^2 - 2y + x^2 + 2 = 0$ can be solved for y in terms of x.

6. (6pts) Consider the following transformations for spherical coordinates:

$$x(\rho, \phi, \theta) = \rho \sin \phi \cos \theta$$
$$y(\rho, \phi, \theta) = \rho \sin \phi \sin \theta$$
$$z(\rho, \phi, \theta) = \rho \cos \phi.$$

When can we solve for (ρ, ϕ, θ) in terms of (x, y, z)?