Calc III: Workshop 2, Fall 2017

Problem 1. Find the point at which the line $x = 3 - t$, $y = 2 + t$, $z = 5t$ intersects the plane $x - y + 2z = 9$.

Problem 2. Find the line of intersection of the planes

$$x + 3y + 2z - 6 = 0, \quad 2x - y + z + 2 = 0.$$

Problem 3. Find the point of intersection (if any) of the line $\frac{x-6}{4} = y + 3 = z$ with the plane $x + 3y + 2z - 6 = 0$.

Problem 4. In general, any four non-coplanar points determine a unique sphere. Find the equation for the sphere determined by the points $(0, 0, 0)$, $(0, 0, 2)$, $(1, -4, 3)$, and $(0, -1, 3)$.

Problem 5. Let S be the sphere with radius 1 centered at $(0, 0, 1)$, and let S^* be S without the "north pole" at the point $(0, 0, 2)$. Let (a, b, c) be an arbitrary point on S^*. Then the line passing through $(0, 0, 2)$ and (a, b, c) intersects the xy-plane at a unique point $(x, y, 0)$. Find the equation for this point $(x, y, 0)$ in terms of (a, b, c). See Figure 1.6.10 in the book.

Remark. This sets up a one-to-one correspondence between points in the plane and points on the sphere with the north pole removed. This is known as *stereographic projection*.