

Final Exam Equation Sheet

Line/plane through \mathbf{p}_0 normal to \mathbf{n} :

$$\mathbf{n} \cdot (\mathbf{p} - \mathbf{p}_0) = 0, \quad \mathbf{p} = (x, y) \text{ or } (x, y, z)$$

Arc length elements:

$$ds = |\mathbf{r}'(t)| dt, \\ d\mathbf{r} = \mathbf{T} ds = \mathbf{r}'(t) dt$$

Line through \mathbf{p}_0 tangent to \mathbf{v} :

$$\mathbf{r}(t) = \mathbf{p}_0 + t\mathbf{v}, \quad t \in \mathbb{R}$$

Surface area elements:

$$dS = |\mathbf{r}_u \times \mathbf{r}_v| du dv, \\ \mathbf{n} dS = \pm \mathbf{r}_u \times \mathbf{r}_v du dv$$

Straight line segment from \mathbf{p}_0 to \mathbf{p}_1 :

$$\mathbf{r}(t) = \mathbf{p}_0 + t(\mathbf{p}_1 - \mathbf{p}_0), \quad 0 \leq t \leq 1.$$

Fundamental Theorem for Line Integrals:

$$f(\mathbf{p}_1) - f(\mathbf{p}_0) = \int_C \nabla f \cdot d\mathbf{r}$$

Curl and divergence:

$$\text{curl } \mathbf{F} = \nabla \times \mathbf{F}, \quad \text{div } \mathbf{F} = \nabla \cdot \mathbf{F}$$

$$\nabla = \frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$$

Stokes'/Green's Theorem:

$$\oint_{\partial S} \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} dS$$

Polar coordinates:

$$(x, y) = (r \cos \theta, r \sin \theta), \\ dA = r dr d\theta$$

Divergence Theorem:

$$\oiint_{\partial E} \mathbf{G} \cdot \mathbf{n} dS = \iiint_E \nabla \cdot \mathbf{G} dV$$

Cylindrical coordinates:

$$(x, y, z) = (r \cos \theta, r \sin \theta, z), \\ dV = r dz dr d\theta$$

Trig identities:

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2} \\ \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

Spherical coordinates:

$$(x, y, z) = (\rho \sin \varphi \cos \theta, \rho \sin \varphi \sin \theta, \rho \cos \varphi), \\ dV = \rho^2 \sin \varphi d\rho d\varphi d\theta$$

Trig values:

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin \theta$	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos \theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0