Calc III: Quiz 1 Solutions, Fall 2018

Problem 1. Let $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - 4\mathbf{k}$.
(a) Find $4\mathbf{a} + 2\mathbf{b}$
(b) Find $|\mathbf{a} - \mathbf{b}|$.

Solution.
(a)

$$2\mathbf{a} + 2\mathbf{b} = 4\langle 4, -3, 2\rangle + 2\langle 2, 0, -4\rangle = \langle 16 + 4, -12 + 0, 8 - 8\rangle = \langle 20, -12, 0\rangle .$$

(b)

$$\mathbf{a} - \mathbf{b} = \langle 4, -3, 2\rangle - \langle 2, 0, -4\rangle = \langle 2, -3, 6\rangle ,$$
$$|\mathbf{a} - \mathbf{b}| = \sqrt{(2)^2 + (-3)^2 + (6)^2} = \sqrt{49} = 7.$$

□

Problem 2. Determine whether the vectors $\mathbf{u}$ and $\mathbf{v}$ are parallel, orthogonal, or neither:
(a) $\mathbf{u} = \langle -5, 4, -2\rangle$, $\mathbf{v} = \langle 3, 4, -1\rangle$
(b) $\mathbf{u} = \langle c, c, c\rangle$, $\mathbf{v} = \langle c, 0, -c\rangle$.

Solution.
(a)

$$\mathbf{u} \cdot \mathbf{v} = (-5)(3) + (4)(4) + (-1)(-2) = -15 + 16 + 2 \neq 0$$

so $\mathbf{u}$ and $\mathbf{v}$ are not orthogonal. They are also not parallel:

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5 & 4 & -2 \\ 3 & 4 & -1 \end{vmatrix} = \langle 4, -11, -32\rangle \neq \langle 0, 0, 0\rangle .$$

(b)

$$\mathbf{u} \cdot \mathbf{v} = c^2 + 0c - c^2 = 0$$

so $\mathbf{u}$ and $\mathbf{v}$ are parallel.

□

Problem 3. Compute the cross product $\langle 2, 3, 0\rangle \times \langle 1, 0, 5\rangle$.

Solution.

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & 0 \\ 1 & 0 & 5 \end{vmatrix} = \mathbf{i}(15 - 0) - \mathbf{j}(10 - 0) + \mathbf{k}(0 - 3) = \langle 15, -10, -3\rangle .$$

□

Problem 4. Find symmetric equations for the line through the points $(0, \frac{1}{2}, 1)$ and $(2, 1, -3)$.

Solution. First we write parametric equations, taking the base point to be $(0, \frac{1}{2}, 1)$ and the direction vector to be the displacement vector

$$\langle 2, 1, -3\rangle - \left\langle 1, \tfrac{1}{2}, 1\right\rangle = \left\langle 2 - 0, 1 - \tfrac{1}{2}, -3 - 1\right\rangle = \left\langle 2, \tfrac{1}{2}, -4\right\rangle$$

which leads to vector/parametric equations

$$\langle x, y, z\rangle = \left\langle 0, \tfrac{1}{2}, 1\right\rangle + t\left\langle 2, \tfrac{1}{2}, -4\right\rangle = \left\langle 2t, \tfrac{1}{2}(1+t), 1 - 4t\right\rangle$$

Solving for t gives the symmetric equations

$$\frac{x}{2} = \frac{2y-1}{1} = \frac{z-1}{-4}.$$

□