Calc III: Quiz 2 Solutions, Fall 2018

Problem 1. Find an equation for the plane through the points $(2, 1, 2)$, $(3, -8, 6)$, and $(-2, -3, 1)$.

Solution. Denoting the points by P_0, P_1, and P_2, respectively, we may form the two difference vectors

$$\mathbf{v}_1 = \vec{P_0P_1} = \langle 1, -9, 4 \rangle, \quad \mathbf{v}_2 = \vec{P_0P_2} = \langle -4, -4, -1 \rangle,$$

the cross product of which gives a normal vector:

$$\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -9 & 4 \\ -4 & -4 & -1 \end{vmatrix} = \langle 25, -15, -40 \rangle .$$

The equation for the plane using normal vector $\mathbf{n}$ and base point P_0 is then

$$\langle 25, -15, -40 \rangle \cdot \langle x - 2, y - 1, z - 2 \rangle = 0, \quad \text{or}$$
$$25(x - 2) - 15(y - 1) - 40(z - 2) = 0.$$

□

Problem 2. Determine whether the planes defined by

$$x + 4y - 3z = 1 \quad \text{and} \quad -3x + 6y + 7z = 0$$

are parallel, orthogonal, or neither.

Solution. We read off the normal vectors from the coefficients of x, y, and z, respectively to get

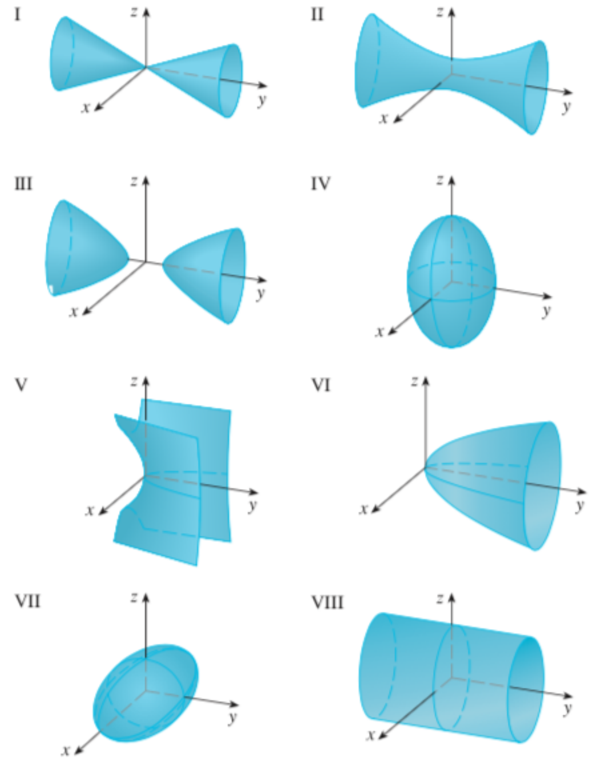
$$\mathbf{n}_1 = \langle 1, 4, -3 \rangle, \qquad \mathbf{n}_2 = \langle -3, 6, 7 \rangle .$$

The planes are orthogonal if and only if these vectors are orthogonal, and we have

$$\mathbf{n}_1 \cdot \mathbf{n}_2 = (1)(-3) + (4)(6) + (-3)(7) = -3 + 24 - 21 = 0,$$

so the planes are orthogonal. □

Problem 3. Match the equation of each surface with one of the graphs pictured below:



(a) $$x^2 - y^2 + z^2 = 1$$

Solution: (II)

(b) $$y = 2x^2 + z^2$$

Solution: (VI)

(c) $$y^2 = x^2 + 2z^2$$

Solution: (I)

(d) $$y = x^2 - z^2$$

Solution: (V)