Calc III: Quiz 3 Solutions, Fall 2018

Problem 1. Define a curve in $\mathbb{R}^3$ by the vector function $\mathbf{r}(t) = \left\langle t^2+1, 4\sqrt{t}, e^{t^2-t}\right\rangle$.

(a) Find the derivative of the vector function $\mathbf{r}(t)$.
(b) Find (parametric or vector) equations for the tangent line to the curve at the point $(2,4,1)$.

Solution.

(a) $\mathbf{r}'(t) = \left\langle 2t, 2t^{-1/2}, (2t-1)e^{t^2-t}\right\rangle$.
(b) The point $(2,4,1) = \mathbf{r}(t)$ for $t=1$, so the curve has tangent vector

$$\mathbf{t}'(1) = \langle 2,2,1\rangle$$

at $(2,4,1)$. The tangent line is given by parametric equation

$$\ell(s) = \langle 2,4,1\rangle + s\,\langle 2,2,1\rangle = \langle 2+2s, 4+2s, 1+s\rangle\,.$$

□

Problem 2. For the function $f(x,y) = x^4y - 2x^3y^2$,

(a) Compute the first partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.
(b) Compute all second partial derivatives of f.

Solution.

(a)

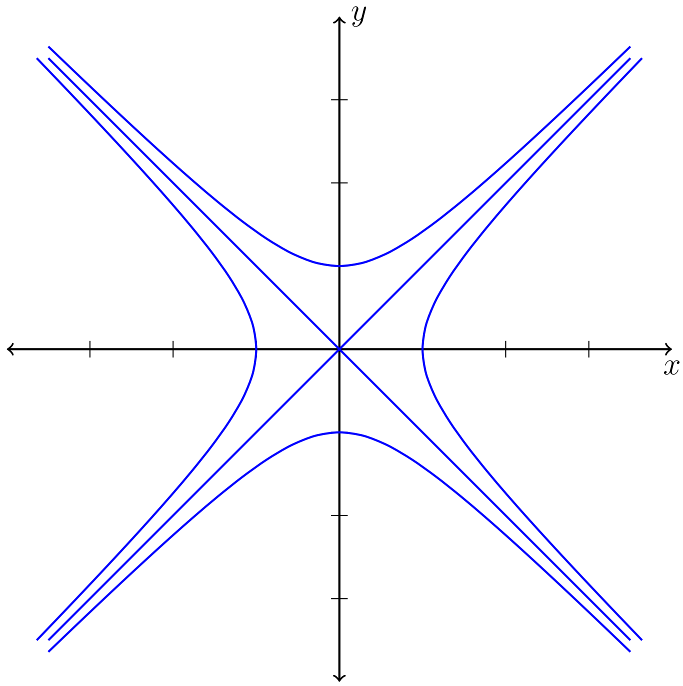
$$\frac{\partial f}{\partial x} = 4x^3y - 6x^2y^2 \quad \text{and} \quad \frac{\partial f}{\partial y} = x^4 - 4x^3y.$$

(b)

$$\frac{\partial^2 f}{\partial x^2} = 12x^2y - 12xy^2 \qquad \frac{\partial^2 f}{\partial y^2} = 4x^3,$$
$$\frac{\partial^2 f}{\partial x \partial y} = 4x^3 - 12x^2y.$$

□

Problem 3. Sketch a contour map of the function $f(x,y) = x^2 - y^2$ with several level curves.



Solution. The level curves $f(x, y) = c$ are hyperbolas, opening out to the left and right for $c > 0$ (meeting the x axis at $x = \pm\sqrt{c}$) and opening up and down for $c < 0$ (meeting the y axis at $y = \pm\sqrt{c}$. For $c = 0$ we have the pair of straight lines $y = \pm x$. $\square$