

Calc III: Quiz 7 Solutions, Fall 2018

Problem 1. Use cylindrical coordinates (r, θ, z) to compute the volume of the solid that is enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 2$.

Solution.

$$\begin{aligned}\text{Vol} &= \int_0^{2\pi} \int_0^1 \int_r^{\sqrt{2-r^2}} r \, dz \, dr \, d\theta \\ &= \int_0^{2\pi} \int_0^1 r\sqrt{2-r^2} - r^2 \, dr \, d\theta \\ &= \int_0^{2\pi} \frac{2^{3/2} - 2}{3} \, d\theta \\ &= \frac{\pi(2^{5/2} - 4)}{3}\end{aligned}$$

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Problem 2. Use spherical coordinates (ρ, θ, ϕ) to evaluate $\iiint_B (x^2 + y^2 + z^2)^2 \, dV$, where B is the ball centered at the origin with radius 5.

Solution.

$$\begin{aligned}\iiint_B (x^2 + y^2 + z^2)^2 \, dV &= \int_0^{2\pi} \int_0^\pi \int_0^5 \rho^5 \sin \phi \, d\rho \, d\phi \, d\theta \\ &= \int_0^{2\pi} d\theta \int_0^\pi \sin \phi \, d\phi \int_0^5 \rho^5 \, d\rho \\ &= (2\pi)(2)\left(\frac{5^6}{6}\right) \\ &= \frac{2\pi 5^6}{3}\end{aligned}$$

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