Calc III: Workshop 13: Course Review, Fall 2018

Problem 1. Sketch the curve with the vector function

$$\mathbf{r}(t) = t\mathbf{i} + \cos \pi t\mathbf{j} + \sin \pi t\mathbf{k}, \quad t \geq 0$$

and find $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$.

Problem 2. Let C be the curve with equations $x = 2 - t^3$, $y = 2t - 1$, $z = \ln t$. Find
(a) the point where C intersects the xz-plane, and
(b) parametric equations for the tangent line at $(1, 1, 0)$.

Problem 3. Find the first partial derivatives of $G(x, y, z) = e^{xz} \sin(y/z)$.

Problem 4. Find an equation of (a) the tangent plane and (b) the normal line of the surface

$$z = 3x^2 - y^2 + 2x$$

at the point $(1, -2, 1)$.

Problem 5. Find the directional derivative of $f(x, y, z) = x^2y + x\sqrt{1 + z}$ at the point $(1, 2, 3)$ in the direction of $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

Problem 6. Find the maximum rate of change of $f(x, y) = x^2y + \sqrt{y}$ at the point $(2, 1)$. In which direction does it occur?

Problem 7. Find the local maximum and minimum values and saddle points of the function

$$f(x, y) = 3xy - x^2y - xy^2$$

Problem 8. Use Lagrange multipliers to find the maximum and minimum values of $f(x, y) = x^2y$, subject to the constraint $x^2 + y^2 = 1$.

Problem 9. Calculate the multiple integral $\iint_D \frac{y}{1+x^2}\, dA$ where D is bounded by $y = \sqrt{x}$, $y = 0$ and $x = 1$.

Problem 10. Compute $\iint_D (x^2 + y^2)^{3/2}\, dA$ where D is the region in the first quadrant bounded by the lines $y = 0$ and $y = \sqrt{3}x$ and the circle $x^2 + y^2 = 9$.

Problem 11. Compute $\iiint_E xy\, dV$, where

$$E = \{(x, y, z) : 0 \leq x \leq 3,\ 0 \leq y \leq x,\ 0 \leq z \leq x + y\}.$$

Problem 12. Compute $\iiint_H z^3\sqrt{x^2 + y^2 + z^2}\, dV$, where H is the solid hemisphere that lies above the xy-plane and has center the origin and radius 1.

Problem 13. Find the volume of the solid under the surface $z = x^2y$ and above the triangle in the xy-plane with vertices $(1, 0)$, $(2, 1)$, and $(4, 0)$.

Problem 14. Find the volume of the solid under the paraboloid $z = x^2 + y^2$ and below the half cone $z = \sqrt{x^2 + y^2}$.

Problem 15. Evaluate the integral $\int_C x\, ds$, where C is the arc of the parabola $y = x^2$ from $(0, 0)$ to $(1, 1)$.

Problem 16. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = e^z\, \mathbf{i} + xz\mathbf{j} + (x + y)\mathbf{k}$ and C is given by $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j} - t\mathbf{k}$, $0 \leq t \leq 1$.

Problem 17. Show that $\mathbf{F}(x, y, z) = e^y\,\mathbf{i} + (xe^y + e^z)\,\mathbf{j} + ye^z\,\mathbf{k}$ is conservative and use this fact to evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ where C is the line segment from $(0, 2, 0)$ to $(4, 0, 3)$.

Problem 18. Use Green's Theorem to evaluate $\int_C \langle x^2y, -xy^2\rangle \cdot d\mathbf{r}$ where C is the triangle with vertices $(0, 0)$, $(1, 0)$ and $(1, 3)$.

Problem 19. Find the surface area of the part of the surface $z = x^2 + 2y$ that lies above the triangle with vertices $(0, 0)$, $(1, 0)$ and $(1, 2)$.

Problem 20. Evaluate the surface/flux integral $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$, where $\mathbf{F}(x, y, z) = \langle x^2,\ xy,\ z\rangle$ and S is the part of the parabolid $z = x^2 + y^2$ below the plane $z = 1$ with upward orientation

Problem 21. Use Stokes' Theorem to evaluate $\iint_S \nabla\times\mathbf{F}\cdot\mathbf{n}\,dS$, where $\mathbf{F}(x, y, z) = x^2yz\,\mathbf{i} + yz^2\,\mathbf{j} + z^3e^{xy}\,\mathbf{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 5$ lying above the plane $z = 1$, oriented upward.

Problem 22. Use Stokes' Theorem to evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = xy\,\mathbf{i} + yz\,\mathbf{j} + zx\,\mathbf{k}$, and C is the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$, oriented counterclockwise as viewed from above.

Problem 23. Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$, where $\mathbf{F}(x, y, z) = \langle x^3, y^3, z^3\rangle$ and S is the total boundary surface of the solid inside the cylinder $x^2 + y^2 = 1$ and between the planes $z = 0$ and $z = 2$.