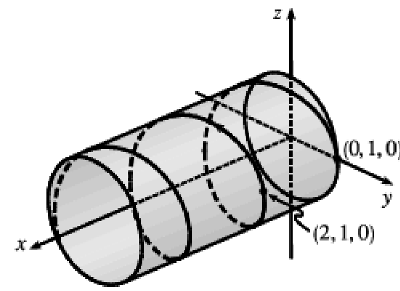
1. (a) The corresponding parametric equations for the curve are $x = t$, $y = \cos \pi t$, $z = \sin \pi t$. Since $y^2 + z^2 = 1$, the curve is contained in a circular cylinder with axis the x-axis. Since $x = t$, the curve is a helix.



(b) $\mathbf{r}(t) = t\,\mathbf{i} + \cos \pi t\,\mathbf{j} + \sin \pi t\,\mathbf{k} \quad \Rightarrow$

$\mathbf{r}'(t) = \mathbf{i} - \pi \sin \pi t\,\mathbf{j} + \pi \cos \pi t\,\mathbf{k} \quad \Rightarrow$

$\mathbf{r}''(t) = -\pi^2 \cos \pi t\,\mathbf{j} - \pi^2 \sin \pi t\,\mathbf{k}$

6. (a) C intersects the xz-plane where $y = 0 \;\Rightarrow\; 2t - 1 = 0 \;\Rightarrow\; t = \frac{1}{2}$, so the point is $\left(2 - \left(\frac{1}{2}\right)^3, 0, \ln \frac{1}{2}\right) = \left(\frac{15}{8}, 0, -\ln 2\right)$.

(b) The curve is given by $\mathbf{r}(t) = \left\langle 2 - t^3, 2t - 1, \ln t\right\rangle$, so $\mathbf{r}'(t) = \left\langle -3t^2, 2, 1/t\right\rangle$. The point $(1, 1, 0)$ corresponds to $t = 1$, so the tangent vector there is $\mathbf{r}'(1) = \langle -3, 2, 1\rangle$. Then the tangent line has direction vector $\langle -3, 2, 1\rangle$ and includes the point $(1, 1, 0)$, so parametric equations are $x = 1 - 3t$, $y = 1 + 2t$, $z = t$.

(c) The normal plane has normal vector $\mathbf{r}'(1) = \langle -3, 2, 1\rangle$ and equation $-3(x - 1) + 2(y - 1) + z = 0$ or $3x - 2y - z = 1$.

16. $G(x, y, z) = e^{xz} \sin(y/z) \quad \Rightarrow \quad G_x = ze^{xz} \sin(y/z),\; G_y = e^{xz} \cos(y/z)(1/z) = (e^{xz}/z) \cos(y/z)$,

$G_z = e^{xz} \cdot \cos(y/z)(-y/z^2) + \sin(y/z) \cdot xe^{xz} = e^{xz}\left[x \sin(y/z) - (y/z^2) \cos(y/z)\right]$

25.

(a) $z_x = 6x + 2 \quad \Rightarrow \quad z_x(1, -2) = 8$ and $z_y = -2y \quad \Rightarrow \quad z_y(1, -2) = 4$, so an equation of the tangent plane is $z - 1 = 8(x - 1) + 4(y + 2)$ or $z = 8x + 4y + 1$.

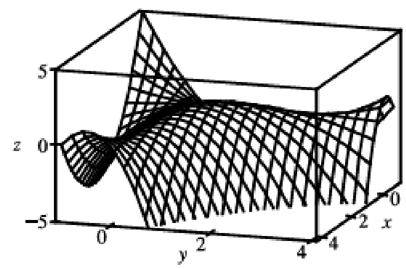
(b) A normal vector to the tangent plane (and the surface) at $(1, -2, 1)$ is $\langle 8, 4, -1\rangle$. Then parametric equations for the normal line there are $x = 1 + 8t$, $y = -2 + 4t$, $z = 1 - t$, and symmetric equations are $\dfrac{x-1}{8} = \dfrac{y+2}{4} = \dfrac{z-1}{-1}$.

46. $\nabla f = \left\langle 2xy + \sqrt{1+z}, x^2, x/\left(2\sqrt{1+z}\right)\right\rangle$, $\nabla f(1, 2, 3) = \left\langle 6, 1, \frac{1}{4}\right\rangle$, $\mathbf{u} = \left\langle \frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right\rangle$. Then $D_{\mathbf{u}}\, f(1, 2, 3) = \frac{25}{6}$.

47. $\nabla f = \left\langle 2xy, x^2 + 1/\left(2\sqrt{y}\right)\right\rangle$, $|\nabla f(2, 1)| = \left|\left\langle 4, \frac{9}{2}\right\rangle\right|$. Thus the maximum rate of change of f at $(2, 1)$ is $\frac{\sqrt{145}}{2}$ in the direction $\left\langle 4, \frac{9}{2}\right\rangle$.

53.

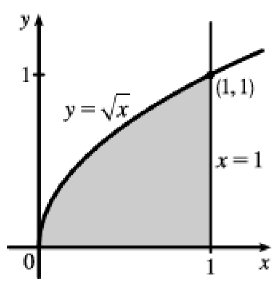
$f(x,y) = 3xy - x^2y - xy^2 \quad \Rightarrow \quad f_x = 3y - 2xy - y^2,\; f_y = 3x - x^2 - 2xy,$
$f_{xx} = -2y$, $f_{yy} = -2x$, $f_{xy} = 3 - 2x - 2y$. Then $f_x = 0$ implies $y(3 - 2x - y) = 0$ so $y = 0$ or $y = 3 - 2x$. Substituting into $f_y = 0$ implies $x(3 - x) = 0$ or $3x(-1 + x) = 0$. Hence the critical points are $(0,0)$, $(3,0)$, $(0,3)$ and $(1,1)$. $D(0,0) = D(3,0) = D(0,3) = -9 < 0$ so $(0,0)$, $(3,0)$, and $(0,3)$ are saddle points. $D(1,1) = 3 > 0$ and $f_{xx}(1,1) = -2 < 0$, so $f(1,1) = 1$ is a local maximum.



59.

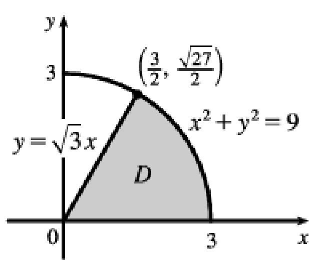
$f(x,y) = x^2y,\; g(x,y) = x^2 + y^2 = 1 \quad \Rightarrow \quad \nabla f = \langle 2xy, x^2 \rangle = \lambda \nabla g = \langle 2\lambda x, 2\lambda y \rangle$. Then $2xy = 2\lambda x$ implies $x = 0$ or $y = \lambda$. If $x = 0$ then $x^2 + y^2 = 1$ gives $y = \pm 1$ and we have possible points $(0, \pm 1)$ where $f(0, \pm 1) = 0$. If $y = \lambda$ then $x^2 = 2\lambda y$ implies $x^2 = 2y^2$ and substitution into $x^2 + y^2 = 1$ gives $3y^2 = 1 \quad \Rightarrow \quad y = \pm \frac{1}{\sqrt{3}}$ and $x = \pm\sqrt{\frac{2}{3}}$. The corresponding possible points are $\left(\pm\sqrt{\frac{2}{3}}, \pm\frac{1}{\sqrt{3}}\right)$. The absolute maximum is $f\left(\pm\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}}\right) = \frac{2}{3\sqrt{3}}$ while the absolute minimum is $f\left(\pm\sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}}\right) = -\frac{2}{3\sqrt{3}}$.

17.



$$\iint_D \frac{y}{1+x^2}\, dA = \int_0^1 \int_0^{\sqrt{x}} \frac{y}{1+x^2}\, dy\, dx = \int_0^1 \frac{1}{1+x^2} \left[\tfrac{1}{2}y^2\right]_{y=0}^{y=\sqrt{x}} dx$$
$$= \frac{1}{2}\int_0^1 \frac{x}{1+x^2}\, dx = \left[\tfrac{1}{4}\ln(1+x^2)\right]_0^1 = \tfrac{1}{4}\ln 2$$

21.



$$\iint_D \left(x^2 + y^2\right)^{3/2} dA = \int_0^{\pi/3} \int_0^3 (r^2)^{3/2} r\, dr\, d\theta$$
$$= \int_0^{\pi/3} d\theta \int_0^3 r^4\, dr = \left[\theta\right]_0^{\pi/3} \left[\tfrac{1}{5}r^5\right]_0^3$$
$$= \frac{\pi}{3}\frac{3^5}{5} = \frac{81\pi}{5}$$

23. $\iiint_E xy\, dV = \int_0^3 \int_0^x \int_0^{x+y} xy\, dz\, dy\, dx = \int_0^3 \int_0^x xy\left[z\right]_{z=0}^{z=x+y} dy\, dx = \int_0^3 \int_0^x xy(x+y)\, dy\, dx$

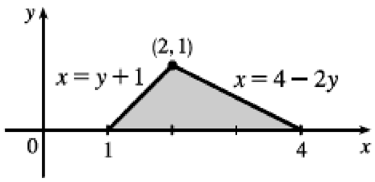
$= \int_0^3 \int_0^x (x^2y + xy^2)\, dy\, dx = \int_0^3 \left[\tfrac{1}{2}x^2y^2 + \tfrac{1}{3}xy^3\right]_{y=0}^{y=x} dx = \int_0^3 \left(\tfrac{1}{2}x^4 + \tfrac{1}{3}x^4\right) dx$

$= \frac{5}{6}\int_0^3 x^4\, dx = \left[\tfrac{1}{6}x^5\right]_0^3 = \frac{81}{2} = 40.5$

28. $\iiint_H z^3\sqrt{x^2+y^2+z^2}\,dV = \int_0^{2\pi}\int_0^{\pi/2}\int_0^1 (\rho^3\cos^3\phi)\rho(\rho^2\sin\phi)\,d\rho\,d\phi\,d\theta$

$$= \int_0^{2\pi} d\theta\, \int_0^{\pi/2}\cos^3\phi\sin\phi\,d\phi\, \int_0^1 \rho^6\,d\rho = 2\pi\left[-\tfrac{1}{4}\cos^4\phi\right]_0^{\pi/2}\left(\tfrac{1}{7}\right) = \tfrac{\pi}{14}$$

30.



$$V = \int_0^1\int_{y+1}^{4-2y}\int_0^{x^2 y} dz\,dx\,dy = \int_0^1\int_{y+1}^{4-2y} x^2 y\,dx\,dy$$
$$= \int_0^1 \tfrac{1}{3}\left[(4-2y)^3 y - (y+1)^3 y\right] dy$$
$$= \int_0^1 3(-y^4+5y^3-11y^2+7y)\,dy = 3\left(-\tfrac{1}{5}+\tfrac{5}{4}-\tfrac{11}{3}+\tfrac{7}{2}\right) = \tfrac{53}{20}$$

34. The paraboloid and the half-cone intersect when $x^2+y^2=\sqrt{x^2+y^2}$, that is when $x^2+y^2=1$ or 0. So

$$V = \iint_{x^2+y^2\le 1}\int_{x^2+y^2}^{\sqrt{x^2+y^2}} dz\,dA = \int_0^{2\pi}\int_0^1\int_{r^2}^{r} r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^1 (r^2-r^3)\,dr\,d\theta = \int_0^{2\pi}\left(\tfrac{1}{3}-\tfrac{1}{4}\right)d\theta = \tfrac{1}{12}(2\pi) = \tfrac{\pi}{6}.$$

2. We can parametrize C by $x=x$, $y=x^2$, $0\le x\le 1$ so

$$\int_C x\,ds = \int_0^1 x\sqrt{1+(2x)^2}\,dx = \tfrac{1}{12}(1+4x^2)^{3/2}\Big]_0^1 = \tfrac{1}{12}\left(5\sqrt{5}-1\right).$$

9. $\mathbf{F}(\mathbf{r}(t)) = e^{-t}\,\mathbf{i} + t^2(-t)\,\mathbf{j} + (t^2+t^3)\,\mathbf{k}$, $\mathbf{r}'(t) = 2t\,\mathbf{i}+3t^2\,\mathbf{j}-\mathbf{k}$ and

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_0^1 (2te^{-t}-3t^5-(t^2+t^3))\,dt = \left[-2te^{-t}-2e^{-t}-\tfrac{1}{2}t^6-\tfrac{1}{3}t^3-\tfrac{1}{4}t^4\right]_0^1 = \tfrac{11}{12}-\tfrac{4}{e}.$$

14. Here curl $\mathbf{F}=\mathbf{0}$, the domain of $\mathbf{F}$ is $\mathbb{R}^3$, and the components of $\mathbf{F}$ have continuous partial derivatives, so $\mathbf{F}$ is conservative. Furthermore $f(x,y,z) = xe^y+ye^z$ is a potential function for $\mathbf{F}$. Then $\int_C \mathbf{F}\cdot d\mathbf{r} = f(4,0,3)-f(0,2,0) = 4-2 = 2.$

17. $\int_C x^2y\,dx - xy^2\,dy = \iint_{x^2+y^2\le 4}\left[\frac{\partial}{\partial x}(-xy^2)-\frac{\partial}{\partial y}(x^2y)\right]dA = \iint_{x^2+y^2\le 4}(-y^2-x^2)\,dA = -\int_0^{2\pi}\int_0^2 r^3\,dr\,d\theta = -8\pi$

25. $z = f(x,y) = x^2+2y$ with $0\le x\le 1$, $0\le y\le 2x$. Thus

$$A(S) = \iint_D \sqrt{1+4x^2+4}\,dA = \int_0^1\int_0^{2x}\sqrt{5+4x^2}\,dy\,dx = \int_0^1 2x\sqrt{5+4x^2}\,dx = \tfrac{1}{6}(5+4x^2)^{3/2}\Big]_0^1 = \tfrac{1}{6}\left(27-5\sqrt{5}\right).$$

30. $z = f(x,y) = x^2+y^2$, $\mathbf{r}_x\times\mathbf{r}_y = -2x\,\mathbf{i}-2y\,\mathbf{j}+\mathbf{k}$ (because of upward orientation) and $\mathbf{F}(\mathbf{r}(x,y))\cdot(\mathbf{r}_x\times\mathbf{r}_y) = -2x^3-2xy^2+x^2+y^2$. Then

$$\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_{x^2+y^2\le 1}(-2x^3-2xy^2+x^2+y^2)\,dA$$
$$= \int_0^1\int_0^{2\pi}(-2r^3\cos^3\theta-2r^3\cos\theta\,\sin^2\theta+r^2)\,r\,dr\,d\theta = \int_0^1 r^3(2\pi)\,dr = \tfrac{\pi}{2}$$

32. $\iint_S \operatorname{curl}\mathbf{F}\cdot d\mathbf{S} = \oint_C \mathbf{F}\cdot d\mathbf{r}$ where C: $\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + \mathbf{k}$, $0 \le t \le 2\pi$, so $\mathbf{r}'(t) = -2\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j}$,

$\mathbf{F}(\mathbf{r}(t)) = 8\cos^2 t\,\sin t\,\mathbf{i} + 2\sin t\,\mathbf{j} + e^{4\cos t\sin t}\,\mathbf{k}$, and $\mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t) = -16\cos^2 t\sin^2 t + 4\sin t\cos t$. Thus

$\oint_C \mathbf{F}\cdot d\mathbf{r} = \int_0^{2\pi}(-16\cos^2 t\,\sin^2 t + 4\sin t\,\cos t)\,dt = \left[-16\left(-\frac{1}{4}\sin t\,\cos^3 t + \frac{1}{16}\sin 2t + \frac{1}{8}t\right) + 2\sin^2 t\right]_0^{2\pi} = -4\pi.$

33. The surface is given by $x + y + z = 1$ or $z = 1 - x - y$, $0 \le x \le 1$, $0 \le y \le 1 - x$ and $\mathbf{r}_x \times \mathbf{r}_y = \mathbf{i} + \mathbf{j} + \mathbf{k}$. Then

$\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_S \operatorname{curl}\mathbf{F}\cdot d\mathbf{S} = \iint_D(-y\,\mathbf{i} - z\,\mathbf{j} - x\,\mathbf{k})\cdot(\mathbf{i} + \mathbf{j} + \mathbf{k})\,dA = \iint_D(-1)\,dA = -(\text{area of } D) = -\frac{1}{2}.$

34. $\iint_S \mathbf{F}\cdot d\mathbf{S} = \iiint_E 3(x^2 + y^2 + z^2)\,dV = \int_0^{2\pi}\int_0^1\int_0^2(3r^2 + 3z^2)\,r\,dz\,dr\,d\theta = 2\pi\int_0^1(6r^3 + 8r)\,dr = 11\pi$