Calc III: Workshop 4, Fall 2018

Problem 1. Let $u = e^{r\theta}\sin\theta$. Compute the partial derivative $\frac{\partial^3 u}{\partial r^2 \partial \theta}$.

Problem 2. Determine whether each of the following functions is a solution to *Laplace's equation* $u_{xx} + u_{yy} = 0$.
(a) $u = x^2 + y^2$
(b) $u = x^2 - y^2$
(c) $u = x^3 - 3xy^2$
(d) $u = \ln\sqrt{x^2 + y^2}$

Problem 3. Is it possible that a function $f(x, y)$ has partial derivatives $f_x(x, y) = x + 4y$ and $f_y(x, y) = 3x - y$?

Problem 4. Find the tangent plane at $(2, -1, -3)$ to the surface

$$z = 3y^2 - 2x^2 + x$$

Problem 5. Find the tangent plane at $(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$ to the surface

$$x^2 + 2y^2 + z^2 = 1.$$

Problem 6. The length ℓ, width w and height h of a box change with time. At a certain instant the dimensions are $\ell = 1$ m and $w = h = 2$ m, and ℓ and w are increasing at a rate of 2 m/s while h is decreasing at a rate of 3 m/s. At that instant find the rates at which the following quantities are changing:
(a) The volume
(b) The surface area
(c) The length of a diagonal

Problem 7.
(a) Given that f is a differentiable function with $f(2, 5) = 6$, $f_x(2, 5) = 1$, and $f_y(2, 5) = -1$, use the linear approximation to estimate $f(2.2, 4.9)$.
(b) Generalize the formula for linear approximations to functions of three variables, find the linear approximation to the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at $(3, 2, 6)$ and use this linear approximation to approximate the number $\sqrt{(3.02)^2 + (1.97)^2 + (5.99)^2}$ (don't use the exact formula).