Calc III: Workshop 7, Fall 2018

Problem 1. A lamina occupies the region in the positive quadrant (where $x \geq 0$ and $y \geq 0$) which lies inside the circle $x^2 + y^2 = 2$ but outside the circle $x^2 + y^2 = 1$. Find the center of mass if the density at any point is inversely proportional to its distance from the origin.

Problem 2. Find the volume of the solid enclosed by the cylinder $x^2 + z^2 = 4$ and the planes $y = -1$ and $y + z = 4$.

Problem 3. Write five other iterated integrals that are equivalent to the iterated integral

$$\int_0^1 \int_y^1 \int_0^y f(x, y, z)\, dz\, dx\, dy.$$

Problem 4. Evaluate the triple integral $\iiint_C (4 + 5x^2yz^2)\, dV$ (try using only geometric interpretation and symmetry), where C is the cylindrical region $x^2 + y^2 \leq 4$, $-2 \leq z \leq 2$.

Problem 5. Find the center of mass of the solid E of constant density, which lies above the xy-plane and below the paraboloid $z = 1 - x^2 - y^2$. [Hint: it may be useful to switch to polar coordinates after integrating in z.]

Problem 6 (Optional bonus)**.** The *simplex of dimension* n is the set T_n of points $(x_1, \ldots, x_n)$ in $\mathbb{R}^n$ bounded by the "hyperplanes" $x_i = 0$ for $i = 1, \ldots, n$ and $x_1 + x_2 + \cdots + x_n = 1$. For $n = 2$ this is a triangle and for $n = 3$ it is a tetrahedron. Compute the 4-dimensional volume of the 4-simplex:

$$\mathrm{Vol}(T_4) = \int\int\int\int 1\, dx_4\, dx_3\, dx_2\, dx_1$$

Formulate a conjecture for the n-dimensional volume of the n-simplex, and try and prove it!