Calc III: Workshop 8, Fall 2018

Problem 1. Find the center of mass of the solid S bounded by the parabolid $z = 4x^2 + 4y^2$ and the plane $z = a$ (where $a > 0$) if S has constant density.

Problem 2. Evaluate $\iiint_E x\, dV$, where E is enclosed by the planes $z = 0$ and $z = x + y + 5$ and the cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

Problem 3. Evaluate the integral

$$\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} \sqrt{x^2 + y^2}\, dz\, dy\, dx$$

by changing to cylindrical coordinates.

Problem 4. Evaluate the integral

$$\int_{0}^{1} \int_{0}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy\, dz\, dy\, dx$$

by changing to spherical coordinates.

Problem 5. Use either cylindrical or spherical coordinates to find the volume of the smaller wedge cut from a sphere of radius a by two planes that intersect along a diameter at an angle of $\pi/6$. [The resulting region looks like an orange slice.]

Problem 6. Evaluate $\iiint_E (x^2 + y^2)\, dV$, where E lies in between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$.

Remark. An identity that may be useful is $\sin^3 \phi = \sin \phi (1 - \cos^2 \phi)$.