Calc III: Workshop 9 (Exam 2 review) , Fall 2018

Problem 1. Find the center of mass $(\overline{x}, \overline{y})$ of the quarter disk

$$Q = \left\{(x,y) : x^2 + y^2 \leq 1,\ 0 \leq x,\ 0 \leq y\right\}$$

assuming Q has unit density ($\delta(x,y) = 1$).

Problem 2. Compute the mass of the region E enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$, assuming its density is given by $\delta(x,y,z) = z$.

Problem 3. For the integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{1-x^2-y^2} f(x,y,z)\, dz\, dy\, dx$$

(a) Change the order of integration from $dz\, dy\, dx$ to $dx\, dy\, dz$, giving the new limits (Hint: for help drawing the 3D region of integration, draw the 2D region indicated by $0 \leq y \leq \sqrt{1-x^2}$, $0 \leq x \leq 1$ along with the surfaces $z = 0$ and $z = 1 - x^2 - y^2$)

(b) Change variables to cylindrical coordinates, giving the new limits in (z, r, θ).

Problem 4. Evaluate $\iiint_E y\, dV$, where E is the solid hemisphere inside $x^2 + y^2 + z^2 = 9$ where $y \geq 0$.

Problem 5. Evaluate the line integral $\int_C x\, ds$, where C is the curve along $y = x^2$ from $(0,0)$ to $(2,4)$.

Problem 6. Let

$$\mathbf{F}(x,y,z) = (2xy+1)z\mathbf{i} + x^2z\mathbf{j} + (x^2y + x + 2z)\mathbf{k}.$$

Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \mathbf{F} \cdot \mathbf{T}\, ds$ where C is the line segment from $(0,0,0)$ to $(1,2,3)$.