Do not write in the boxes immediately below.

problem	1	2	3	4	5	6	7	8	9	10	11	12	total
points													

Math 2321 Final Exam

December 12, 2012

Instructor's name______________________________ Your name______________________________

Answers from your calculator, without supporting work, are worth zero points.

1) Let $\mathbf{v} = (3, -2, -2)$ and $\mathbf{w} = (-4, 1, 1)$.

a) (2 points) Calculate $\cos\theta$, where θ is the angle between the two vectors v and w.

b) (3 points) Calculate the orthogonal projection of $\mathbf{v}$ onto $\mathbf{w}$.

c) (4 points) Give a vector equation for the line which passes through the point $(1, -2, 4)$ and is orthogonal to the vectors $\mathbf{v}$ and $\mathbf{w}$.

2) (2 points each) The volume $V = V(p, T)$ of a specific quantity of a gas is a function of the pressure p and the temperature T. Suppose that V is measured in cubic feet, T is in °F, and p is in lb/in^2. Suppose, further, that $V(24, 500) = 23.69$

a) Thinking physically about the situation, should $\partial V/\partial p$ at $(24, 500)$ be positive or negative? Explain briefly.

b) Suppose that you can reliably measure V when p changes by as small an increment as 2 lb/in^2 and/or when T changes by as small an increment as 20°F.

If you're going to take a measurement of V at just one new point (p_1, T_1), where $p_1 \geq 24$ and $T_1 \geq 500$, what should you pick for (p_1, T_1), in order to have the data that you need to obtain a good approximation of $\partial V/\partial p$ at $(24, 500)$? (You are **not** being asked to produce the approximation in this part of the problem; you are just supposed to supply the point (p_1, T_1).)

c) Assume that $V(p_1, T_1) = 21.86$, where (p_1, T_1) is the point that you supplied above. What approximation do you obtain for $\partial V/\partial p$ at $(p, T) = (24, 500)$?

d) Assume that $\partial V/\partial T = 0.0255$ ft^3/°F, when $(p, T) = (24, 500)$. Combining this with the data above, what do you obtain for the linearization of the function V at $(p, T) = (24, 500)$?

3) The pressure P, in atmospheres (atm), produced by oxygen in a bottle, with a piston, is given by

$$P = \frac{nRT}{V - 0.03n} - 1.4\left(\frac{n}{V}\right)^2,$$

where n is the number of moles of gas, T is the temperature in Kelvins (K), V is the volume in liters, and R is the gas constant 0.082 L-atm/mol-K. (Note that the constants 0.03 and 1.4 in the formula are assumed to have the appropriate units.)

a) (5 points) Find $\frac{\partial P}{\partial V}$ and $\frac{\partial P}{\partial T}$.

b) (5 points) Suppose that n is held constant at $n = 5$. Also, suppose, when $V = 5$ liters and $T = 300$ K, that V is increasing at a rate of 0.5 liters/s and T is increasing at a rate of 10 K/s. Find the rate of change of P, with respect to time, at this moment.

4) Suppose that the xy-plane is being heated, and let $T = T(x, y)$ denote the temperature, in °C, at the point (x, y), where x and y are measured in meters. Suppose that, at the point $(5, 10)$, the temperature decreases at a rate of 0.3°C per meter in the $\mathbf{i}$ direction, and increases at a rate of 0.4°C per meter in the $\mathbf{j}$ direction.

a) (2 points) What is the gradient vector of the temperature at $(5, 10)$? (If you cannot do this part, make up an answer, so that you can obtain credit in the parts below.)

b) (3 points) What is the rate of change of T, with respect to distance, in the direction of $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$? What is the physical meaning of this number?

c) (2 points) What is the greatest rate of change of temperature, with respect to distance, at $(5, 10)$?

d) (2 points) An *isotherm* is a curve along which the temperature is constant. Heat flows along curves which are perpendicular to the isotherms, moving from high temperatures to low temperatures. What is the direction (as a unit vector) in which the heat moves at $(5, 10)$?

5) (8 points) Find all critical points of the function $f(x, y) = x^2 + 50y^2 + x^2y$. For each critical point, determine whether it corresponds to a local maximum value, a local minimum value, or a saddle point of f.

6) Consider the iterated integral $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\,dx\,dy$ and let $\int\int_R \sqrt{x^3+1}\,dA$ be the corresponding double integral.

a) (2 points) Sketch the region of integration R.

b) (3 points) Give an iterated integral for $\int\int_R \sqrt{x^3+1}\,dA$ in which the order of integration is reversed.

c) (3 points) Evaluate your iterated integral from part b).

7) (8 points) Consider the "cup" formed by the part of the paraboloid $z = x^2 + y^2$ that lies above the disk of radius 3, centered at the origin, in the xy-plane. If x, y, and z are measured in meters, what volume of liquid can this cup hold?

8) (8 points) Find the mass of the region between the upper hemispheres of radius 2 meters and 4 meters, centered at the origin, if the density of the region is given by $\delta(\rho, \phi, \theta) = \rho$ kg/m^3, where ρ is the distance from a point in the region to the origin.

9) (8 points) Give an iterated integral for computing the surface area of the part of the paraboloid $z = 1 - x^2 - y^2$, where $z \geq 0$. Your iterated integral must include limits of integration.

DO NOT EVALUATE THIS INTEGRAL.

10) (8 points) Find the work done by the force field $\mathbf{F} = 2xy\,\mathbf{i} + (x^2 + y^2)\,\mathbf{j} = (2xy,\, x^2 + y^2)$ Newtons, where x and y are in meters, as it moves a particle from $(4, -2)$ to $(4, 2)$ along the curve where $x = y^2$.

11) (8 points) Consider the vector field in space $\mathbf{F}(x, y, z) = \left(y\,\mathrm{e}^{z^2},\ z\ln(1 + x^2),\ -2\right)$. Let M be the portion of the graph $z = 5 - x^2 - y^2$ which sits above the plane $z = 0$, and orient M outwards/upwards. Compute the flux of $\mathbf{F}$ through M.

12) In Maxwell's theory of electrodynamics, the magnetic field $\mathbf{B}(x, y, z)$ throughout space is given by the curl of the "vector potential" $\mathbf{A}(x, y, z)$, i.e., $\mathbf{B} = \vec{\nabla} \times \mathbf{A}$.

Suppose that $\mathbf{A}(x, y, z) = \left(\sin z,\, 2x - z^2,\, x\,\mathrm{e}^y\right)$.

a) (3 points) Compute the magnetic field $\mathbf{B}(x, y, z)$.

b) (5 points) Let M be the portion of the sphere $x^2 + y^2 + z^2 = 4$ which sits above the plane $z = 1$. Compute the flux of $\mathbf{B}$ through M, oriented upwards.