Do not write in the boxes immediately below.

problem	1	2	3	4	5	6	7	8	9	10	11	12	total
points													

Math 2321 Final Exam

December 12, 2013

Instructor's name________________________ Your name________________________

Please check that you have 9 different pages.

Answers from your calculator, without supporting work, are worth zero points.

1) A charge distribution on a plane is creating an electric field. The electrical potential $P(x,y)$ measures the potential energy of a unit point charge due to its position in the field. The function is given by $P(x,y) = \frac{2}{\sqrt{(x+2)^2+(y-1)^2}}$.

a) (4 points) Find the gradient vector of the potential at $(1,5)$.

b) (4 points) An equipotential line is a curve on our plate along which the potential is constant. What is an equation for the tangent line of the equipotential line passing through $(1,5)$?

2) (8 points) Find the critical points of $f(x, y) = x^3 + 8y^3 - 6xy$, verify that each critical point is non-degenerate, and determine what type of critical point it is.

3) (10 points) Suppose that a cardboard box is to be constructed with **no** top and a volume of 4000 cubic inches. Suppose that the cardboard for the bottom costs 5 cents per square inch, while the cardboard for the sides costs 1 cent per square inch. Find the dimensions of the box which minimize the cost of the cardboard required.

4) (8 points) For the following sum of integrals, $\int_0^1 \int_0^{x^2} y\, dy\, dx \; + \; \int_1^2 \int_0^{2-x} y\, dy\, dx$, reverse the order of integration and evaluate the resulting integral using this new order.

5) (10 points) Let S be the solid region which is bounded on the sides and top by the planes where $x = 0$ and $x + z = 1$ and on the bottom by the parabolic cylinder where $z = y^2 - 1$. Compute the volume of S.

6) (8 points) Let S be the solid region in the 1st octant (i.e., where $x \geq 0$, $y \geq 0$, and $z \geq 0$) in $\mathbb{R}^3$ which is contained within the sphere where $x^2 + y^2 + z^2 = 16$, bounded by the cones where $z = \sqrt{x^2 + y^2}$ and $z\sqrt{3} = \sqrt{x^2 + y^2}$, and bounded by the planes with equations $y = x$ and $y\sqrt{3} = x$. Find the volume of S.

7) (8 points) Find the mass of the solid right circular cylinder where $-2 \leq z \leq 2$ and $x^2 + y^2 \leq 4$, if the density of the solid region is given by $\delta(x, y) = x^2 + y^2$ kg/m^3. Here x, y, and z are in meters.

8) (8 points) Let $f(x, y, z) = x^2 + y^3 + z^4$ and $\mathbf{F} = \vec{\nabla} f$. Find the line integral of $\mathbf{F}$ along the oriented curve, consisting of four line segments, which go from $(1, 0, 0)$ to $(1, 2, 5)$, then from $(1, 2, 5)$ to $(2, -3, 7)$, then from $(2, -3, 7)$ to $(-4, 6, -7)$, and then from $(-4, 6, -7)$ to $(0, 0, 1)$.

9) (8 points) Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = (x^2 - y,\, y^2 + x)$ and C is the circle of radius 5 centered at $(1, 1)$ and oriented clockwise.

10) Consider the parameterization $\mathbf{r}(u, v) = (u^2 + v, u + v, uv)$, where $1 \le u \le 2$ and $0 \le v \le 1$, and let M be the surface parameterized by $\mathbf{r}$.

a) (3 points) We want to orient M by using $\dfrac{\mathbf{r}_u \times \mathbf{r}_v}{|\mathbf{r}_u \times \mathbf{r}_v|}$ for the positive direction. Show that this is possible by showing that $\mathbf{r}_u \times \mathbf{r}_v \ne \mathbf{0}$ (the zero vector) provided that $1 \le u \le 2$ and $0 \le v \le 1$.

b) (5 points) Orient M as in part (a). Consider the vector field $\mathbf{F}(x, y, z) = (y - x,\ z,\ 0)$. Set up, but **do not evaluate** an iterated integral, in terms of u and v for the flux integral $\int\int_M \mathbf{F} \cdot \mathbf{n}\, dS$ of $\mathbf{F}$ through M. You should "simplify" your iterated integral by evaluating any dot products or cross products, until all that remains is an iterated integral of a polynomial in the variables u and v.

11) (8 points) Let $\mathbf{V}(x, y, z) = (P(x, y, z),\, Q(x, y, z),\, R(x, y, z))$ be a continuously differentiable velocity vector field, in m/s, where x, y, and z are measured in meters.

Suppose it is known that $Q(x, y, z) = y^2 + e^z$ and $R(x, y, z) = z^3 + \sin x$, in meters per second. Furthermore, suppose that the flux of V is measured through a large number of closed surfaces, with the result that the flux is always 0 m^3/s. From this, assume that the flux is always 0 m^3/s through every reasonably nice closed surface. Give one possible function P that would make this true.

12) Suppose that $\mathbf{F}(x,y,z) = (ze^x,\, 2x + y^3 + 7z,\, e^x + 3y + \sin z)$ is a force field on $\mathbb{R}^3$, measured in Newtons, where x, y, and z are measured in meters.

a) (2 points) Calculate the curl, $\vec{\nabla} \times \mathbf{F}$, of $\mathbf{F}$.

We continue to use the vector field $\mathbf{F}$ from above. Suppose that M is a surface, with boundary ∂M, in $\mathbb{R}^3$ about which you have the following data: M is contained in the plane P where $x + 2y + 3z = 6$, the area of M is 7 square meters, and the positive direction on M is chosen to point upwards, away from the origin (i.e., is chosen to have a positive z component).

b) (1 point) To give the boundary ∂M the orientation that is compatible with the orientation on M, should you orient ∂M clockwise or counterclockwise, if you are looking downwards from above the plane P, towards the origin?

c) (4 points) Giving ∂M its compatible orientation, calculate $\int_{\partial M} \mathbf{F} \cdot d\mathbf{r}$.

(You have enough data to answer this. Hint: What is a normal vector to the plane?)

d) (1 point) Physically, what does $\int_{\partial M} \mathbf{F} \cdot d\mathbf{r}$ give you?