Do not write in the boxes immediately below.

problem	1	2	3	4	5	6	7	8	9	total
points										

Calculus 3, Final Exam

April 24, 2014

Instructor's name ________________ Your name ________________

Show all your work in the space provided. Use the back page if necessary. No credit for unjustified answers. You may use a calculator to check your answers but must do all calculations by hand. Formula sheet is allowed.

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(1) (10 points) Suppose that the temperature measured in degrees Celsius at each point of a metal plate is given by $T(x, y) = e^x \cos y + e^y \cos x$, where x and y are given in meters.

a) (4 points) In what direction does the temperature increase most rapidly at the point $(0, 0)$?

ANSWER: ________________

b) (3 points) What is the rate of increase in that direction?

ANSWER: ________________

c) (3 points) In what direction does the temperature decrease most rapidly at the point $(0, 0)$?

ANSWER: ________________

(2) (10 points) Find the critical points of the function

$$f(x, y) = 2xy^2 - 8y^2 - x^2$$

and classify them as local maxima, minima or saddle points.

ANSWER: ____

(3) (12 points) Use Lagrange multipliers to find the critical points of the restriction of

$$f(x, y, z) = x + 2y + 3z - 12$$

to the surface given by $x^2 + 2y^2 + 3z^2 = 1$.

ANSWER: ______________________________

(4) (10 points) Calculate the volume below the graph of $z = 12 + x^2 - y^2$ and above the filled-in triangle in the xy-plane which has vertices $(0, 0, 0)$, $(1, 0, 0)$, and $(1, 2, 0)$.

ANSWER: ______________________________________

(5) (10 points) A solid occupies the region which is in the 1-st octant (where $x \geq 0$, $y \geq 0$, $z \geq 0$), inside the half-cone given by $z = \sqrt{x^2 + y^2}$, and between the spheres of radius 3 and 5 centered at the origin. Suppose that x, y, and z are measured in meters, and that the density function is $\delta(x, y, z) = \dfrac{1}{x^2 + y^2 + z^2}$ kg/m^3. Calculate the mass of the solid.

ANSWER: ______________________________

(6) (12 points) Given $\vec{F}(x,y) = \big(\sin(x)e^{x^2} - 8y^3, 8x^3 - y^4\ln(1+y^2)\big)$, find the line integral of $\vec{F}$ along C, if C is the oriented curve that starts at $(-1,0)$ goes along the semi-circle of radius 1 centered at the origin with $y \geq 0$, and then goes back along the x-axis to $(-1,0)$.

ANSWER: ____________________

(7) (12 points) Given $\vec{F}(x, y) = \big(1 + ye^x + y, x + e^x\big)$, using the Fundamental Theorem of Line Integrals, find the line integral of $\vec{F}$ along C, if C is the oriented curve that consists of the line segment from $(0, 0)$ to $(13, 1)$, followed by the line segment from $(13, 1)$ to $(1, 1)$.

ANSWER: ______________________________

(8) (12 points) Let C be the curve that is the intersection of the plane given by $z = x$ with the cylinder given by $x^2 + y^2 = 4$, oriented clockwise as viewed from above.

a) (5 points) Parametrize the surface of the plane $z = x$ that is bounded by the curve C.

ANSWER: ______________________

b) (7 points) Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y, z) = \big(2z, x, 3\big)$.

ANSWER: ______________________

(9) (12 points) Evaluate the flux integral $\iint\limits_S \vec{F} \cdot \vec{n}\, dS$ where

$$\vec{F} = \left(z^2 + xy^2, yx^2, y + z\right)$$

and S is the surface given by $z = x^2 + y^2$ that is above the disk of radius 1 in the xy-plane centered at the origin. The surface is oriented by the upward normal.

ANSWER: ______________________________