MATH 3150 – HOMEWORK 4

Update 9/30: There was a typo in Problem 4.

Problem 1. Prove the converse in Proposition 1.5.5.(i). In other words, Suppose that if x_n is a sequence in $\mathbb{R}$ which is bounded below, and $a \in \mathbb{R}$ has the following properties:

(a) For all $\epsilon > 0$, there exists an N such that $x_n > a - \epsilon$ for all $n \geq N$.
(b) For all $\epsilon > 0$ and for all $M \in \mathbb{N}$, there exists an $n \geq M$ such that $x_n < a + \epsilon$ (in other words, for any ϵ there are infinitely many $x_n < a + \epsilon$),

Then prove that $a = \liminf x_n$.

Problem 2 (p.98, # 9.)**.** Let x_n be a bounded sequence in $\mathbb{R}$ and let $y_n = (-1)^n x_n$. Show that $\limsup y_n \leq \limsup |x_n|$. Need we have equality? Formulate a similar inequality for $\liminf$.

Problem 3 (p.64 # 5.)**.** Find the equation of the line through $(1, 1, 1)$ and $(2, 3, 4)$. Is this line a linear subspace?

Problem 4 (p. 98 # 14.)**.**

(a) Prove *Lagrange's identity*

$$\left(\sum_{i=1}^{n} x_i y_i\right)^2 = \left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} y_i^2\right) - \sum_{1 \leq i < j \leq n} (x_i y_j - x_j y_i)^2.$$

and use this to give another proof of the Cauchy-Scwharz inequality.

(b) Show that

$$\left(\sum_{i=1}^{n} (x_i + y_i)^2\right)^{1/2} \leq \left(\sum_{i=1}^{n} x_i^2\right)^{1/2} + \left(\sum_{i=1}^{n} y_i^2\right)^{1/2}.$$