Celebrating Singularity: A conference in honor of Richard Melrose

Abstracts

Tanya Christiansen: Eigenvalues and resonances of Schrödinger operators near zero in dimension 2

We compute asymptotics of eigenvalues approaching the bottom of the continuous spectrum, and associated resonances, for Schrödinger operators in dimension two. We distinguish persistent eigenvalues, which have associated resonances, from disappearing ones, which do not. We illustrate the significance of this distinction by computing corresponding scattering phase asymptotics and numerical Breit–Wigner peaks. While we concentrate on the case of the circular well for illustrative and computational purposes, we also prove some of our results for more general potentials, using recent results on low-energy resonance expansions. This talk is based on joint work with Kiril Datchev and Colton Griffin, and is part of a larger project with K. Datchev.

Julian Edwards: On the null-controllability of the structurally damped beam equation

Let Δ be the Dirichlet Laplacian on the interval $(0, \pi)$. The null controllability properties of the equation

$$u_{tt} + \Delta^2 u + \rho(\Delta)^\alpha u_t = F(x,t)$$

are studied. Let $T > 0$, and assume initial conditions $(u^0, u^1) \in Dom(\Delta) \times L^2(0,\pi)$. We first prove finite dimensional null control results: Suppose $F(x,t) = f^1(t)h^1(x) + f^2(t)h^2(x)$ with h^1, h^2 given functions. For $\alpha < 3/2$, we prove that there exist $h^1, h^2 \in L^2(0,\pi)$ such that for any (u^0, u^1), there exist L^2 null controls (f^1, f^2). For $\alpha < 1$ and $\rho < 2$, we prove null controllability with $f^2 = 0$ and h^1 belonging to a large class of functions. For $\alpha \geq 3/2$, we prove null controllability generally fails. Our second set of results pertains to $F(x,t) = \chi_\Omega(x) f(x,t)$, with Ω any open subset of $(0,\pi)$. For any $\alpha \in (0, 3/2)$, we prove there exists a null control $f \in L^2(\Omega \times (0,T))$ To prove our main results, we use the Fourier method to rewrite the control problems as moment problems. These are then solved by constructing biorthogonal sets to the associated exponential families. We then consider null-controllability using Dirichlet controls. Here, a main difficulty is computing the adjoint of Δ^α given non-homogeneous boundary conditions. Partial results are achieved using heat kernel estimates due to E.B. Davies.

Rafe Mazzeo: Geometric flows of singular spaces

I will describe a selection of recent results about nonlinear geometric flows (Ricci, mean curvature, etc.) on stratified spaces. Geometric microlocal methods have played a very useful role. Amongst results I'll describe are an older one with Rubinstein and Sesum about Ricci/Yamabe flow on spaces with conic singularities, and more recent work with Pluda and Saez about flowing networks and foams of surfaces.

Frédéric Rochon: Warped quasi-asymptotically conical Calabi-Yau metrics

We will explain how to construct new examples of complete Calabi-Yau manifolds of maximal volume growth on certain smoothings of Cartesian products of Calabi-Yau cones. A description of the geometry at infinity will be given in terms of a compactification by a manifold with corners obtained through a suitable sequence of blow-ups. A key analytical step in the construction of these Calabi-Yau metrics is to derive good mapping properties of the Laplacian on some suitable weighted Hölder spaces. Our methods also produce Calabi-Yau metrics with an isolated conical singularity modelled on a Calabi-Yau cone distinct from the tangent cone at infinity, in particular yielding a transition behavior between different Calabi-Yau cones as conjectured by Yang Li. This is used to exhibit many examples where the tangent cone at infinity does not uniquely specify a Calabi-Yau metric with exact Kaehler form. This is a joint work with Ronan Conlon.

Antonio Sá Baretto: Nonlinear geometric optics and inverse problems for the wave equation

We use nonlinear geometric optics techniques to show that one can recover a semilinear potential from scattering experiments.

Fang Wang: Fractional GJMS operators on the conformal infinity of Poincare-Einstein manifolds

The Fractional GJMS operators are the renormalized scattering operators on the conformal infinity of Poincare-Einstein Manifolds. They provide a bridge to transfer the information of interior Einstein geometry to the boundary conformal geometry. These operators form a one-parameter family of conformally invariant operators. In this talk, I will first introduce some properties for this family, including the comparison theorem for fractional Yamabe constants of different orders and the positive mass theorem for them. Then I will also present some geometric inverse scattering theorems via this this family of operators.

Jared Wunsch: Caustics of weakly Lagrangian distributions

A semiclassical sequence of eigenstates of an integrable system can be said to have definite values of the integrals of the motion if their semiclassical wavefront set concentrates on a single Arnol'd-Liouville torus. This can happen at different rates as $h \downarrow 0$, with the fastest corresponding to the case of a semiclassical Lagrangian distribution. We will explore how the sup-norm along such a sequence varies according to different rates of concentration along a Lagrangian manifold; the answer involves both the rate of concentration and the form of the projection of the Lagrangian to the base. This is joint work with Sean Gomes.

Xuwen Zhu: Degeneration of hyperbolic surfaces and spectral gaps for large genus

The study of "small" eigenvalues of the Laplacian on hyperbolic surfaces has a long history and has recently seen many developments. In this talk I will focus on the recent work (joint with Yunhui Wu and Haohao Zhang) on the higher spectral gaps, where we study the differences of consecutive eigenvalues up to λ_{2g-2} for genus g hyperbolic surfaces. We show that the supremum of such spectral gaps over the moduli space has infimum limit at least $1/4$ as genus goes to infinity. The analysis relies on previous joint works with Richard Melrose on degenerating hyperbolic surfaces.

Maciej Zworski: Global lower bounds for QNM for Schwarzschild black holes

We prove that the number of quasinormal modes (QNM) for Schwarzschild and Schwarzschild–de Sitter (SdS) black holes in a disc is bounded from below by the third power of the radius of the disc (in fact we give asymptotics in conic neighbourhoods of the real axis). This shows that Jezequel's recent upper bound (in the case of SdS) is sharp. The argument is an application of a spectral asymptotics result for non-self-adjoint operators which provides a finer description of QNM, explaining the emergence of a distorted lattice and generalizing the lattice structure in strips described by Sá Barreto and the speaker. The topic will also allow us to review Richard Melrose's foundational work on the counting of scattering poles (QNM are the name for them in General Relativity). The talk is based on joint work with M. Hitrik.