MATH 3150 – HOMEWORK 1

Problem 1. Prove the following proposition.

Proposition. *In an ordered field, the following properties hold:*

(i) **Unique identities.** *If* $a + x = a$ *for every* a*, then* $x = 0$*. If* $a \cdot x = a$ *for every* a*, then* $x = 1$*.*
(ii) **Unique inverses.** *If* $a + x = 0$*, then* $x = -a$*. If* $ax = 1$*, then* $x = a^{-1}$*.*
(iii) **No divisors of zero**. *If* $xy = 0$*, then* $x = 0$ *or* $y = 0$*.*
(iv) **Cancellation for addition.** *If* $a + x = b + x$ *then* $a = b$*. If* $a + x \leq b + x$*, then* $a \leq b$*.*
(v) **Cancellation for multiplication.** *If* $ax = bx$ *and* $x \neq 0$*, then* $a = b$*. If* $ax \geq bx$ *and* $x > 0$*, then* $a \geq b$*.*
(vi) $0 \cdot x = 0$ *for every* x*.*
(vii) $-(-x) = x$ *for every* x*.*
(viii) $-x = (-1) \cdot x$ *for every* x*.*
(ix) *If* $x \neq 0$*, then* $x^{-1} \neq 0$ *and* $(x^{-1})^{-1} = x$*.*
(x) *If* $x \neq 0$ *and* $y \neq 0$*, then* $xy \neq 0$ *and* $(xy)^{-1} = x^{-1}y^{-1}$*.*
(xi) *If* $x \leq y$ *and* $0 \leq z$*, then* $xz \leq yz$*. If* $x \leq y$ *and* $z \leq 0$*, then* $yz \leq xz$*.*
(xii) *If* $x \leq 0$ *and* $y \leq 0$*, then* $xy \geq 0$*. If* $x \leq 0$ *and* $0 \leq y$*, then* $xy \leq 0$*.*
(xiii) $0 < 1$*.*
(xiv) *For any* x*,* $x^2 \geq 0$*.*

Hint: You can prove these in any order you wish. Once you've proved a particular property, you can use it in your proof of a later property.

Problem 2. Give an example of a field with only three elements. Prove that it cannot be made into an ordered field.

Problem 3. Show that $3^n/n!$ converges to 0.

Problem 4. Let $x_n = \sqrt{n^2 + 1} - n$. Compute $\lim_{n \to \infty} x_n$.