MATH 3150 — HOMEWORK 7

Problem 1 (p. 236, #43)**.** Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be continuous and set $F(x) = \int_0^{x^2} f(y)\,dy$. Prove that $F'(x) = 2xf(x^2)$. State a more general theorem.

Problem 2 (p. 316, #2)**.** Determine which of the following sequences converge (pointwise or uniformly) as $k \to \infty$. Check the continuity of the limit in each case.
(a) $(\sin x)/k$ on $\mathbb{R}$
(b) $1/(kx+1)$ on $(0,1)$
(c) $x/(kx+1)$ on $(0,1)$
(d) $x/(1+kx^2)$ on $\mathbb{R}$
(e) $(1, (\cos x)/k^2)$, as functions from $\mathbb{R}$ to $\mathbb{R}^2$

Problem 3 (p. 318, #18)**.** Give an example of a sequence of discontinuous functions f_k converging *uniformly* to a limit function f that is continuous.

Problem 4 (p. 272, #4)**.** Let

$$f_n(x) = \frac{1}{n}\frac{nx}{1+nx}, \quad 0 \le x \le 1.$$

Show that $f_n \longrightarrow 0$ in $\mathcal{C}([0,1];\mathbb{R})$.

Problem 5 (p. 272, #5)**.** Let f_k be a convergent sequence in $\mathcal{C}_b(A;\mathbb{R}^m)$. Prove that $\{f_k : k = 1, 2, \ldots\}$ is bounded in $\mathcal{C}_b(A;\mathbb{R}^m)$. Is it closed? [Hint: Don't overthink this!]